

### I. TÍNH CÁC GÓC CỦA TAM GIÁC

**Bài 201:** Tính các góc của  $\Delta ABC$  nếu :

$$\sin(B + C) + \sin(C + A) + \cos(A + B) = \frac{3}{2} \quad (*)$$

Do  $A + B + C = \pi$

Nên:  $(*) \Leftrightarrow \sin A + \sin B - \cos C = \frac{3}{2}$

$$\Leftrightarrow 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2} - \left( 2 \cos^2 \frac{C}{2} - 1 \right) = \frac{3}{2}$$

$$\Leftrightarrow 2 \cos \frac{C}{2} \cos \frac{A-B}{2} - 2 \cos^2 \frac{C}{2} = \frac{1}{2}$$

$$\Leftrightarrow 4 \cos^2 \frac{C}{2} - 4 \cos \frac{C}{2} \cos \frac{A-B}{2} + 1 = 0$$

$$\Leftrightarrow \left( 2 \cos \frac{C}{2} - \cos \frac{A-B}{2} \right)^2 + 1 - \cos^2 \frac{A-B}{2} = 0$$

$$\Leftrightarrow \left( 2 \cos \frac{C}{2} - \cos \frac{A-B}{2} \right)^2 + \sin^2 \frac{A-B}{2} = 0$$

$$\Leftrightarrow \begin{cases} 2 \cos \frac{C}{2} = \cos \frac{A-B}{2} \\ \sin \frac{A-B}{2} = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} 2 \cos \frac{C}{2} = \cos 0 = 1 \\ \frac{A-B}{2} = 0 \end{cases} \Leftrightarrow \begin{cases} \frac{C}{2} = \frac{\pi}{3} \\ A = B \end{cases}$$

$$\Leftrightarrow \begin{cases} A = B = \frac{\pi}{6} \\ C = \frac{2\pi}{3} \end{cases}$$

**Bài 202:** Tính các góc của  $\Delta ABC$  biết:

$$\cos 2A + \sqrt{3}(\cos 2B + \cos 2C) + \frac{5}{2} = 0 \quad (*)$$

Ta có:  $(*) \Leftrightarrow 2 \cos^2 A - 1 + 2\sqrt{3}[\cos(B+C)\cos(B-C)] + \frac{5}{2} = 0$

$$\begin{aligned} &\Leftrightarrow 4 \cos^2 A - 4\sqrt{3} \cos A \cdot \cos(B - C) + 3 = 0 \\ &\Leftrightarrow \left[ 2 \cos A - \sqrt{3} \cos(B - C) \right]^2 + 3 - 3 \cos^2(B - C) = 0 \\ &\Leftrightarrow \left[ 2 \cos A - \sqrt{3} \cos(B - C) \right]^2 + 3 \sin^2(B - C) = 0 \\ &\Leftrightarrow \begin{cases} \sin(B - C) = 0 \\ \cos A = \frac{\sqrt{3}}{2} \cos(B - C) \end{cases} \Leftrightarrow \begin{cases} B - C = 0 \\ \cos A = \frac{\sqrt{3}}{2} \end{cases} \\ &\Leftrightarrow \begin{cases} A = 30^\circ \\ B = C = 75^\circ \end{cases} \end{aligned}$$

**Bài 203:** Chứng minh  $\Delta ABC$  có  $C = 120^\circ$  nếu :

$$\sin A + \sin B + \sin C - 2 \sin \frac{A}{2} \cdot \sin \frac{B}{2} = 2 \sin \frac{C}{2} \quad (*)$$

Ta có

$$\begin{aligned} (*) &\Leftrightarrow 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2} + 2 \sin \frac{C}{2} \cos \frac{C}{2} = 2 \sin \frac{A}{2} \sin \frac{B}{2} + 2 \sin \frac{C}{2} \\ &\Leftrightarrow 2 \cos \frac{C}{2} \cos \frac{A-B}{2} + 2 \sin \frac{C}{2} \cos \frac{C}{2} = 2 \cos \frac{A+B}{2} + 2 \sin \frac{A}{2} \sin \frac{B}{2} \\ &\Leftrightarrow \cos \frac{C}{2} \left( \cos \frac{A-B}{2} + \sin \frac{C}{2} \right) = \cos \frac{A}{2} \cdot \cos \frac{B}{2} \\ &\Leftrightarrow \cos \frac{C}{2} \left[ \cos \frac{A-B}{2} + \cos \frac{A+B}{2} \right] = \cos \frac{A}{2} \cos \frac{B}{2} \\ &\Leftrightarrow 2 \cos \frac{C}{2} \cos \frac{A}{2} \cos \frac{B}{2} = \cos \frac{A}{2} \cos \frac{B}{2} \\ &\Leftrightarrow \cos \frac{C}{2} = \frac{1}{2} \quad (\text{do } \cos \frac{A}{2} > 0 \text{ và } \cos \frac{B}{2} > 0 \text{ vì } 0 < \frac{A}{2}; \frac{B}{2} < \frac{\pi}{2}) \\ &\Leftrightarrow C = 120^\circ \end{aligned}$$

**Bài 204:** Tính các góc của  $\Delta ABC$  biết số đo 3 góc tạo cấp số cộng và

$$\sin A + \sin B + \sin C = \frac{3 + \sqrt{3}}{2}$$

Không làm mất tính chất tổng quát của bài toán giả sử  $A < B < C$

Ta có:  $A, B, C$  tạo 1 cấp số cộng nên  $A + C = 2B$

Mà  $A + B + C = \pi$  nên  $B = \frac{\pi}{3}$

Lúc đó:  $\sin A + \sin B + \sin C = \frac{3 + \sqrt{3}}{2}$

$$\Leftrightarrow \sin A + \sin \frac{\pi}{3} + \sin C = \frac{3 + \sqrt{3}}{2}$$

$$\Leftrightarrow \sin A + \sin C = \frac{3}{2}$$

$$\Leftrightarrow 2 \sin \frac{A+C}{2} \cos \frac{A-C}{2} = \frac{3}{2}$$

$$\Leftrightarrow 2 \cos \frac{B}{2} \cos \frac{A-C}{2} = \frac{3}{2}$$

$$\Leftrightarrow 2 \cdot \left(\frac{\sqrt{3}}{2}\right) \cos \frac{A-C}{2} = \frac{3}{2}$$

$$\Leftrightarrow \cos \frac{C-A}{2} = \frac{\sqrt{3}}{2} = \cos \frac{\pi}{6}$$

Do  $C > A$  nên  $\Delta ABC$  có:

$$\begin{cases} \frac{C-A}{2} = \frac{\pi}{6} \\ C+A = \frac{2\pi}{3} \\ B = \frac{\pi}{3} \end{cases} \Leftrightarrow \begin{cases} C = \frac{\pi}{2} \\ A = \frac{\pi}{6} \\ B = \frac{\pi}{3} \end{cases}$$

**Bài 205:** Tính các góc của  $\Delta ABC$  nếu

$$\begin{cases} b^2 + c^2 \leq a^2 & (1) \end{cases}$$

$$\begin{cases} \sin A + \sin B + \sin C = 1 + \sqrt{2} & (2) \end{cases}$$

Áp dụng định lý hàm cosin:  $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

Do (1):  $b^2 + c^2 \leq a^2$  nên  $\cos A \leq 0$

Do đó:  $\frac{\pi}{2} \leq A < \pi \Leftrightarrow \frac{\pi}{4} \leq \frac{A}{2} < \frac{\pi}{2}$

Vậy  $\cos \frac{A}{2} \leq \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$  (\*)

$$\begin{aligned} \text{Mặt khác: } \sin A + \sin B + \sin C &= \sin A + 2 \sin \frac{B+C}{2} \cos \frac{B-C}{2} \\ &= \sin A + 2 \cos \frac{A}{2} \cos \frac{B-C}{2} \\ &\leq 1 + 2 \left(\frac{\sqrt{2}}{2}\right) \cdot 1 \quad \left(\text{do (*) và } \cos \frac{B-C}{2} \leq 1\right) \end{aligned}$$

Mà  $\sin A + \sin B + \sin C = 1 + \sqrt{2}$  do (2)

$$\text{Dấu “=” tại (2) xảy ra} \Leftrightarrow \begin{cases} \sin A = 1 \\ \cos \frac{A}{2} = \frac{\sqrt{2}}{2} \\ \cos \frac{B-C}{2} = 1 \end{cases} \Leftrightarrow \begin{cases} A = \frac{\pi}{2} \\ B = C = \frac{\pi}{4} \end{cases}$$

**Bài 206:** (Đề thi tuyển sinh Đại học khối A, năm 2004)

Cho  $\Delta ABC$  không tù thỏa điều kiện

$$\cos 2A + 2\sqrt{2} \cos B + 2\sqrt{2} \cos C = 3 \quad (*)$$

Tính ba góc của  $\Delta ABC$

\* **Cách 1:** Đặt  $M = \cos 2A + 2\sqrt{2} \cos B + 2\sqrt{2} \cos C - 3$

$$\text{Ta có: } M = 2\cos^2 A + 4\sqrt{2} \cos \frac{B+C}{2} \cos \frac{B-C}{2} - 4$$

$$\Leftrightarrow M = 2\cos^2 A + 4\sqrt{2} \sin \frac{A}{2} \cos \frac{B-C}{2} - 4$$

$$\text{Do } \sin \frac{A}{2} > 0 \text{ và } \cos \frac{B-C}{2} \leq 1$$

$$\text{Nên } M \leq 2\cos^2 A + 4\sqrt{2} \sin \frac{A}{2} - 4$$

$$\text{Mặt khác: } \Delta ABC \text{ không tù nên } 0 < A \leq \frac{\pi}{2}$$

$$\Rightarrow 0 \leq \cos A \leq 1$$

$$\Rightarrow \cos^2 A \leq \cos A$$

$$\text{Do đó: } M \leq 2\cos A + 4\sqrt{2} \sin \frac{A}{2} - 4$$

$$\Leftrightarrow M \leq \left(1 - 2\sin^2 \frac{A}{2}\right) + 4\sqrt{2} \sin \frac{A}{2} - 4$$

$$\Leftrightarrow M \leq -4\sin^2 \frac{A}{2} + 4\sqrt{2} \sin \frac{A}{2} - 2$$

$$\Leftrightarrow M \leq -2\left(\sqrt{2} \sin \frac{A}{2} - 1\right)^2 \leq 0$$

Do giả thiết (\*) ta có  $M=0$

$$\text{Vậy: } \begin{cases} \cos^2 A = \cos A \\ \cos \frac{B-C}{2} = 1 \\ \sin \frac{A}{2} = \frac{1}{\sqrt{2}} \end{cases} \Leftrightarrow \begin{cases} A = 90^\circ \\ B = C = 45^\circ \end{cases}$$

\* **Cách 2:** (\*)  $\Leftrightarrow \cos 2A + 2\sqrt{2} \cos B + 2\sqrt{2} \cos C - 3 = 0$

$$\Leftrightarrow \cos^2 A + 2\sqrt{2} \cos \frac{B+C}{2} \cos \frac{B-C}{2} - 2 = 0$$

$$\Leftrightarrow (\cos^2 A - \cos A) + \cos A + 2\sqrt{2} \sin \frac{A}{2} \cos \frac{B-C}{2} - 2 = 0$$

$$\Leftrightarrow \cos A (\cos A - 1) + \left(1 - 2 \sin^2 \frac{A}{2}\right) + 2\sqrt{2} \sin \frac{A}{2} \cos \frac{B-C}{2} - 2 = 0$$

$$\Leftrightarrow \cos A (\cos A - 1) - \left(\sqrt{2} \sin \frac{A}{2} - \cos \frac{B-C}{2}\right)^2 - \left(1 - \cos^2 \frac{B-C}{2}\right) = 0$$

$$\Leftrightarrow \cos A (\cos A - 1) - \left(\sqrt{2} \sin \frac{A}{2} - \cos \frac{B-C}{2}\right)^2 - \sin^2 \frac{B-C}{2} = 0 (*)$$

Do  $\Delta ABC$  không tù nên  $\cos A \geq 0$  và  $\cos A - 1 < 0$

Vậy vế trái của (\*) luôn  $\leq 0$

$$\text{Dấu "=" xảy ra} \Leftrightarrow \begin{cases} \cos A = 0 \\ \sqrt{2} \sin \frac{A}{2} = \cos \frac{B-C}{2} \\ \sin \frac{B-C}{2} = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} A = 90^\circ \\ B = C = 45^\circ \end{cases}$$

**Bài 207:** Chứng minh  $\Delta ABC$  có ít nhất 1 góc  $60^\circ$  khi và chỉ khi

$$\frac{\sin A + \sin B + \sin C}{\cos A + \cos B + \cos C} = \sqrt{3} (*)$$

Ta có:

$$(*) \Leftrightarrow (\sin A - \sqrt{3} \cos A) + (\sin B - \sqrt{3} \cos B) + (\sin C - \sqrt{3} \cos C) = 0$$

$$\Leftrightarrow \sin\left(A - \frac{\pi}{3}\right) + \sin\left(B - \frac{\pi}{3}\right) + \sin\left(C - \frac{\pi}{3}\right) = 0$$

$$\Leftrightarrow 2 \sin\left(\frac{A+B}{2} - \frac{\pi}{3}\right) \cos \frac{A-B}{2} + \sin\left(C - \frac{\pi}{3}\right) = 0$$

$$\Leftrightarrow 2 \sin\left[\left(\frac{\pi}{2} - \frac{C}{2}\right) - \frac{\pi}{3}\right] \cos \frac{A-B}{2} + 2 \sin\left(\frac{C}{2} - \frac{\pi}{6}\right) \cos\left(\frac{C}{2} - \frac{\pi}{6}\right) = 0$$

$$\Leftrightarrow 2 \sin\left(\frac{C}{2} - \frac{\pi}{6}\right) \left[-\cos \frac{A-B}{2} + \cos\left(\frac{C}{2} - \frac{\pi}{6}\right)\right] = 0$$

$$\Leftrightarrow \sin\left(\frac{C}{2} - \frac{\pi}{6}\right) = 0 \vee \cos \frac{A-B}{2} = \cos\left(\frac{C}{2} - \frac{\pi}{6}\right) = \cos\left(\frac{\pi}{3} - \frac{A+B}{2}\right)$$

$$\Leftrightarrow \frac{C}{2} = \frac{\pi}{6} \vee \frac{A-B}{2} = \frac{\pi}{3} - \frac{A+B}{2} \vee \frac{-A+B}{2} = \frac{\pi}{3} - \frac{A+B}{2}$$

$$\Leftrightarrow C = \frac{\pi}{3} \vee A = \frac{\pi}{3} \vee B = \frac{\pi}{3}$$

**Bài 208:** Cho  $\Delta ABC$  và  $V = \cos^2 A + \cos^2 B + \cos^2 C - 1$ . Chứng minh:

a/ Nếu  $V = 0$  thì  $\Delta ABC$  có một góc vuông

b/ Nếu  $V < 0$  thì  $\Delta ABC$  có ba góc nhọn

c/ Nếu  $V > 0$  thì  $\Delta ABC$  có một góc tù

$$\text{Ta có: } V = \frac{1}{2}(1 + \cos 2A) + \frac{1}{2}(1 + \cos 2B) + \cos^2 C - 1$$

$$\Leftrightarrow V = \frac{1}{2}(\cos 2A + \cos 2B) + \cos^2 C$$

$$\Leftrightarrow V = \cos(A + B) \cdot \cos(A - B) + \cos^2 C$$

$$\Leftrightarrow V = -\cos C \cdot \cos(A - B) + \cos^2 C$$

$$\Leftrightarrow V = -\cos C [\cos(A - B) + \cos(A + B)]$$

$$\Leftrightarrow V = -2 \cos C \cos A \cos B$$

Do đó:

a/  $V = 0 \Leftrightarrow \cos A = 0 \vee \cos B = 0 \vee \cos C = 0$

$\Leftrightarrow \Delta ABC \perp$  tại A hay  $\Delta ABC \perp$  tại B hay  $\Delta ABC \perp$  tại C

b/  $V < 0 \Leftrightarrow \cos A \cdot \cos B \cdot \cos C > 0$

$\Leftrightarrow \Delta ABC$  có ba góc nhọn

(vì trong 1 tam giác không thể có nhiều hơn 1 góc tù nên không có trường hợp có 2 cos cùng âm)

c/  $V > 0 \Leftrightarrow \cos A \cdot \cos B \cdot \cos C < 0$

$\Leftrightarrow \cos A < 0 \vee \cos B < 0 \vee \cos C < 0$

$\Leftrightarrow \Delta ABC$  có 1 góc tù.

## II. TAM GIÁC VUÔNG

**Bài 209:** Cho  $\Delta ABC$  có  $\cotg \frac{B}{2} = \frac{a+c}{b}$

Chứng minh  $\Delta ABC$  vuông

Ta có:  $\cotg \frac{B}{2} = \frac{a+c}{b}$

$$\Leftrightarrow \frac{\cos \frac{B}{2}}{\sin \frac{B}{2}} = \frac{2R \sin A + 2R \sin C}{2R \sin B} = \frac{\sin A + \sin C}{\sin B}$$

$$\Leftrightarrow \frac{\cos \frac{B}{2}}{\sin \frac{B}{2}} = \frac{2 \sin \frac{A+C}{2} \cdot \cos \frac{A-C}{2}}{2 \sin \frac{B}{2} \cdot \cos \frac{B}{2}}$$

$$\Leftrightarrow \cos^2 \frac{B}{2} = \cos \frac{B}{2} \cdot \cos \frac{A-C}{2} \quad (\text{do } \sin \frac{B}{2} > 0)$$

$$\Leftrightarrow \cos \frac{B}{2} = \cos \frac{A-C}{2} \quad (\text{do } \cos \frac{B}{2} > 0)$$

$$\Leftrightarrow \frac{B}{2} = \frac{A-C}{2} \vee \frac{B}{2} = \frac{C-A}{2}$$

$$\Leftrightarrow A = B + C \vee C = A + B$$

$$\Leftrightarrow A = \frac{\pi}{2} \vee C = \frac{\pi}{2}$$

$\Leftrightarrow \Delta ABC$  vuông tại A hay  $\Delta ABC$  vuông tại C

**Bài 210:** Chứng minh  $\Delta ABC$  vuông tại A nếu

$$\frac{b}{\cos B} + \frac{c}{\cos C} = \frac{a}{\sin B \sin C}$$

Ta có:

$$\frac{b}{\cos B} + \frac{c}{\cos C} = \frac{a}{\sin B \sin C}$$

$$\Leftrightarrow \frac{2R \sin B}{\cos B} + \frac{2R \sin C}{\cos C} = \frac{2R \sin A}{\sin B \sin C}$$

$$\Leftrightarrow \frac{\sin B \cos C + \sin C \cos B}{\cos B \cdot \cos C} = \frac{\sin A}{\sin B \sin C}$$

$$\Leftrightarrow \frac{\sin(B+C)}{\cos B \cdot \cos C} = \frac{\sin A}{\sin B \sin C}$$

$$\Leftrightarrow \cos B \cos C = \sin B \sin C \text{ (do } \sin A > 0)$$

$$\Leftrightarrow \cos B \cdot \cos C - \sin B \cdot \sin C = 0$$

$$\Leftrightarrow \cos(B+C) = 0$$

$$\Leftrightarrow B+C = \frac{\pi}{2}$$

$$\Leftrightarrow \Delta ABC \text{ vuông tại A}$$

**Bài 211:** Cho  $\Delta ABC$  có:

$$\cos \frac{A}{2} \cdot \cos \frac{B}{2} \cdot \cos \frac{C}{2} - \sin \frac{A}{2} \cdot \sin \frac{B}{2} \cdot \sin \frac{C}{2} = \frac{1}{2} \quad (*)$$

Chứng minh  $\Delta ABC$  vuông

Ta có:

$$(*) \Leftrightarrow \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2} = \frac{1}{2} + \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$$

$$\Leftrightarrow \frac{1}{2} \left[ \cos \frac{A+B}{2} + \cos \frac{A-B}{2} \right] \cos \frac{C}{2} = \frac{1}{2} - \frac{1}{2} \left[ \cos \frac{A+B}{2} - \cos \frac{A-B}{2} \right] \sin \frac{C}{2}$$

$$\Leftrightarrow \left[ \sin \frac{C}{2} + \cos \frac{A-B}{2} \right] \cos \frac{C}{2} = 1 - \left[ \sin \frac{C}{2} - \cos \frac{A-B}{2} \right] \sin \frac{C}{2}$$

$$\Leftrightarrow \sin \frac{C}{2} \cos \frac{C}{2} + \cos \frac{A-B}{2} \cos \frac{C}{2} = 1 - \sin^2 \frac{C}{2} + \cos \frac{C}{2} = 1 - \sin^2 \frac{C}{2} + \cos \frac{A-B}{2} \sin \frac{C}{2}$$

$$\Leftrightarrow \sin \frac{C}{2} \cos \frac{C}{2} + \cos \frac{A-B}{2} \cos \frac{C}{2} = \cos^2 \frac{C}{2} + \cos \frac{A-B}{2} \sin \frac{C}{2}$$

$$\Leftrightarrow \cos \frac{C}{2} \left[ \sin \frac{C}{2} - \cos \frac{C}{2} \right] = \cos \frac{A-B}{2} \left[ \sin \frac{C}{2} - \cos \frac{C}{2} \right]$$

$$\Leftrightarrow \left[ \sin \frac{C}{2} - \cos \frac{C}{2} \right] \left[ \cos \frac{C}{2} - \cos \frac{A-B}{2} \right] = 0$$

$$\Leftrightarrow \sin \frac{C}{2} = \cos \frac{C}{2} \vee \cos \frac{C}{2} = \cos \frac{A-B}{2}$$

$$\Leftrightarrow \operatorname{tg} \frac{C}{2} = 1 \vee \frac{C}{2} = \frac{A-B}{2} \vee \frac{C}{2} = \frac{B-A}{2}$$

$$\Leftrightarrow \frac{C}{2} = \frac{\pi}{4} \vee A = B + C \vee B = A + C$$

$$\Leftrightarrow C = \frac{\pi}{2} \vee A = \frac{\pi}{2} \vee B = \frac{\pi}{2}$$

**Bài 212:** Chứng minh  $\Delta ABC$  vuông nếu:  
 $3(\cos B + 2 \sin C) + 4(\sin B + 2 \cos C) = 15$

Do bất đẳng thức Bunhiacốpki ta có:

$$3 \cos B + 4 \sin B \leq \sqrt{9 + 16} \sqrt{\cos^2 B + \sin^2 B} = 15$$

và  $6 \sin C + 8 \cos C \leq \sqrt{36 + 64} \sqrt{\sin^2 C + \cos^2 C} = 10$

nên:  $3(\cos B + 2 \sin C) + 4(\sin B + 2 \cos C) \leq 15$

Dấu “=” xảy ra  $\Leftrightarrow \begin{cases} \frac{\cos B}{3} = \frac{\sin B}{4} \\ \frac{\sin C}{6} = \frac{\cos C}{8} \end{cases} \Leftrightarrow \begin{cases} \operatorname{tg} B = \frac{4}{3} \\ \operatorname{cotg} C = \frac{4}{3} \end{cases}$

$$\Leftrightarrow \operatorname{tg} B = \operatorname{cotg} C$$

$$\Leftrightarrow B + C = \frac{\pi}{2}$$

$$\Leftrightarrow \Delta ABC \text{ vuông tại } A.$$

**Bài 213:** Cho  $\Delta ABC$  có:  $\sin 2A + \sin 2B = 4 \sin A \cdot \sin B$   
 Chứng minh  $\Delta ABC$  vuông.

Ta có:  $\sin 2A + \sin 2B = 4 \sin A \cdot \sin B$

$$\Leftrightarrow 2 \sin(A + B) \cos(A - B) = -2 [\cos(A + B) - \cos(A - B)]$$

$$\Leftrightarrow \cos(A + B) = [1 - \sin(A + B)] \cos(A - B)$$

$$\Leftrightarrow -\cos C = [1 - \sin C] \cos(A - B)$$

$$\Leftrightarrow -\cos C(1 + \sin C) = (1 - \sin^2 C) \cdot \cos(A - B)$$

$$\Leftrightarrow -\cos C(1 + \sin C) = \cos^2 C \cdot \cos(A - B)$$

$$\Leftrightarrow \cos C = 0 \text{ hay } -(1 + \sin C) = \cos C \cdot \cos(A - B) \quad (*)$$

$$\Leftrightarrow \cos C = 0$$

(Do  $\sin C > 0$  nên  $-(1 + \sin C) < -1$ )

Mà  $\cos C \cdot \cos(A - B) \geq -1$ . Vậy (\*) vô nghiệm.)

Do đó  $\Delta ABC$  vuông tại C

### III. TAM GIÁC CÂN

**Bài 214:** Chứng minh nếu  $\Delta ABC$  có  $\operatorname{tg}A + \operatorname{tg}B = 2 \operatorname{cotg} \frac{C}{2}$

thì là tam giác cân.

$$\text{Ta có: } \operatorname{tg}A + \operatorname{tg}B = 2 \operatorname{cotg} \frac{C}{2}$$

$$\Leftrightarrow \frac{\sin(A+B)}{\cos A \cdot \cos B} = \frac{2 \cos \frac{C}{2}}{\sin \frac{C}{2}}$$

$$\Leftrightarrow \frac{\sin C}{\cos A \cdot \cos B} = \frac{2 \cos \frac{C}{2}}{\sin \frac{C}{2}}$$

$$\Leftrightarrow \frac{2 \sin \frac{C}{2} \cos \frac{C}{2}}{\cos A \cos B} = \frac{2 \cos \frac{C}{2}}{\sin \frac{C}{2}}$$

$$\Leftrightarrow \sin^2 \frac{C}{2} = \cos A \cdot \cos B \quad \left( \text{do } \cos \frac{C}{2} > 0 \right)$$

$$\Leftrightarrow \frac{1}{2}(1 - \cos C) = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$$

$$\Leftrightarrow 1 - \cos C = -\cos C + \cos(A-B)$$

$$\Leftrightarrow \cos(A-B) = 1$$

$$\Leftrightarrow A = B$$

$$\Leftrightarrow \Delta ABC \text{ cân tại } C.$$

**Bài 215:** Chứng minh  $\Delta ABC$  cân nếu:

$$\sin \frac{A}{2} \cdot \cos^3 \frac{B}{2} = \sin \frac{B}{2} \cdot \cos^3 \frac{A}{2}$$

$$\text{Ta có: } \sin \frac{A}{2} \cdot \cos^3 \frac{B}{2} = \sin \frac{B}{2} \cdot \cos^3 \frac{A}{2}$$

$$\Leftrightarrow \left( \frac{\sin \frac{A}{2}}{\cos \frac{A}{2}} \right) \frac{1}{\cos^2 \frac{A}{2}} = \left( \frac{\sin \frac{B}{2}}{\cos \frac{B}{2}} \right) \frac{1}{\cos^2 \frac{B}{2}}$$

$$\left( \text{do } \cos \frac{A}{2} > 0 \text{ và } \cos \frac{B}{2} > 0 \right)$$

$$\Leftrightarrow \operatorname{tg} \frac{A}{2} \left( 1 + \operatorname{tg}^2 \frac{A}{2} \right) = \operatorname{tg} \frac{B}{2} \left( 1 + \operatorname{tg}^2 \frac{B}{2} \right)$$

$$\Leftrightarrow \operatorname{tg}^3 \frac{A}{2} - \operatorname{tg}^3 \frac{B}{2} + \operatorname{tg} \frac{A}{2} - \operatorname{tg} \frac{B}{2} = 0$$

$$\Leftrightarrow \left( \operatorname{tg} \frac{A}{2} - \operatorname{tg} \frac{B}{2} \right) \left[ 1 + \operatorname{tg}^2 \frac{A}{2} + \operatorname{tg}^2 \frac{B}{2} + \operatorname{tg} \frac{A}{2} \cdot \operatorname{tg} \frac{B}{2} \right] = 0 \quad (*)$$

$$\Leftrightarrow \operatorname{tg} \frac{A}{2} = \operatorname{tg} \frac{B}{2} \quad (\text{vì } 1 + \operatorname{tg}^2 \frac{A}{2} + \operatorname{tg}^2 \frac{B}{2} + \operatorname{tg} \frac{A}{2} \operatorname{tg} \frac{B}{2} > 0)$$

$$\Leftrightarrow A = B$$

$$\Leftrightarrow \Delta ABC \text{ cân tại } C$$

**Bài 216:** Chứng minh  $\Delta ABC$  cân nếu:

$$\frac{\cos^2 A + \cos^2 B}{\sin^2 A + \sin^2 B} = \frac{1}{2} (\cotg^2 A + \cotg^2 B) \quad (*)$$

Ta có:

$$(*) \Leftrightarrow \frac{\cos^2 A + \cos^2 B}{\sin^2 A + \sin^2 B} = \frac{1}{2} \left( \frac{1}{\sin^2 A} + \frac{1}{\sin^2 B} - 2 \right)$$

$$\Leftrightarrow \frac{\cos^2 A + \cos^2 B}{\sin^2 A + \sin^2 B} + 1 = \frac{1}{2} \left( \frac{1}{\sin^2 A} + \frac{1}{\sin^2 B} \right)$$

$$\Leftrightarrow \frac{2}{\sin^2 A + \sin^2 B} = \frac{1}{2} \left( \frac{1}{\sin^2 A} + \frac{1}{\sin^2 B} \right)$$

$$\Leftrightarrow 4 \sin^2 A \sin^2 B = (\sin^2 A + \sin^2 B)^2$$

$$\Leftrightarrow 0 = (\sin^2 A - \sin^2 B)$$

$$\Leftrightarrow \sin A = \sin B$$

Vậy  $\Delta ABC$  cân tại C

**Bài 217:** Chứng minh  $\Delta ABC$  cân nếu:

$$a + b = \operatorname{tg} \frac{C}{2} (\operatorname{atg} A + \operatorname{btg} B) \quad (*)$$

Ta có:  $a + b = \operatorname{tg} \frac{C}{2} (\operatorname{atg} A + \operatorname{btg} B)$

$$\Leftrightarrow (a + b) \cotg \frac{C}{2} = \operatorname{atg} A + \operatorname{btg} B$$

$$\Leftrightarrow a \left[ \operatorname{tg} A - \cotg \frac{C}{2} \right] + b \left[ \operatorname{tg} B - \cotg \frac{C}{2} \right] = 0$$

$$\Leftrightarrow a \left[ \operatorname{tg} A - \operatorname{tg} \frac{A+B}{2} \right] + b \left[ \operatorname{tg} B - \operatorname{tg} \frac{A+B}{2} \right] = 0$$

$$\Leftrightarrow \frac{a \sin \frac{A-B}{2}}{\cos A \cdot \cos \frac{A+B}{2}} + \frac{b \sin \frac{B-A}{2}}{\cos B \cdot \cos \frac{A+B}{2}} = 0$$

$$\Leftrightarrow \sin \frac{A-B}{2} = 0 \text{ hay } \frac{a}{\cos A} - \frac{b}{\cos B} = 0$$

$$\Leftrightarrow A = B \text{ hay } \frac{2R \sin A}{\cos A} = \frac{2R \sin B}{\cos B}$$

$$\Leftrightarrow A = B \text{ hay } \operatorname{tg} A = \operatorname{tg} B \Leftrightarrow \Delta ABC \text{ cân tại } C$$

#### IV. NHẬN DẠNG TAM GIÁC

**Bài 218:** Cho  $\Delta ABC$  thỏa:  $a \cos B - b \cos A = a \sin A - b \sin B$  (\*)  
 Chứng minh  $\Delta ABC$  vuông hay cân

Do định lý hàm sin:  $a = 2R \sin A, b = 2R \sin B$

$$\text{Nên (*)} \Leftrightarrow 2R \sin A \cos B - 2R \sin B \cos A = 2R (\sin^2 A - \sin^2 B)$$

$$\Leftrightarrow \sin A \cos B - \sin B \cos A = \sin^2 A - \sin^2 B$$

$$\Leftrightarrow \sin(A-B) = \frac{1}{2}(1 - \cos 2A) - \frac{1}{2}(1 - \cos 2B)$$

$$\Leftrightarrow \sin(A-B) = \frac{1}{2}[\cos 2B - \cos 2A]$$

$$\Leftrightarrow \sin(A-B) = -[\sin(A+B)\sin(B-A)]$$

$$\Leftrightarrow \sin(A-B)[1 - \sin(A+B)] = 0$$

$$\Leftrightarrow \sin(A-B) = 0 \vee \sin(A+B) = 1$$

$$\Leftrightarrow A = B \vee A + B = \frac{\pi}{2}$$

vậy  $\Delta ABC$  vuông hay cân tại C

Cách khác

$$\sin A \cos B - \sin B \cos A = \sin^2 A - \sin^2 B$$

$$\Leftrightarrow \sin(A-B) = (\sin A + \sin B)(\sin A - \sin B)$$

$$\Leftrightarrow \sin(A-B) = \left(2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}\right) \left(2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}\right)$$

$$\Leftrightarrow \sin(A-B) = \sin(A+B)\sin(A-B)$$

$$\Leftrightarrow \sin(A-B) = 0 \vee \sin(A+B) = 1$$

$$\Leftrightarrow A = B \vee A + B = \frac{\pi}{2}$$

**Bài 219**  $\Delta ABC$  là tam giác gì nếu  
 $(a^2 + b^2)\sin(A-B) = (a^2 - b^2)\sin(A+B)$  (\*)

Ta có: (\*)

$$\Leftrightarrow (4R^2 \sin^2 A + 4R^2 \sin^2 B)\sin(A-B) = 4R^2(\sin^2 A - \sin^2 B)\sin(A+B)$$

$$\Leftrightarrow \sin^2 A[\sin(A-B) - \sin(A+B)] + \sin^2 B[\sin(A-B) + \sin(A+B)] = 0$$

$$\Leftrightarrow 2\sin^2 A \cos A \sin(-B) + 2\sin^2 B \sin A \cos B = 0$$

$$\Leftrightarrow -\sin A \cos A + \sin B \cos B = 0 \text{ (do } \sin A > 0 \text{ và } \sin B > 0 \text{)}$$

$$\Leftrightarrow \sin 2A = \sin 2B$$

$$\Leftrightarrow 2A = 2B \vee 2A = \pi - 2B$$

$$\Leftrightarrow A = B \vee A + B = \frac{\pi}{2}$$

Vậy  $\Delta ABC$  cân tại C hay  $\Delta ABC$  vuông tại C.

**Bài 220:**  $\Delta ABC$  là tam giác gì nếu:

$$\begin{cases} a^2 \sin 2B + b^2 \sin 2A = 4ab \cos A \sin B & (1) \\ \sin 2A + \sin 2B = 4 \sin A \sin B & (2) \end{cases}$$

Ta có:

$$(1) \Leftrightarrow 4R^2 \sin^2 A \sin 2B + 4R^2 \sin^2 B \sin 2A = 16R^2 \sin A \sin^2 B \cos A$$

$$\Leftrightarrow \sin^2 A \sin 2B + \sin^2 B \sin 2A = 4 \sin A \sin^2 B \cos A$$

$$\Leftrightarrow 2 \sin^2 A \sin B \cos B + 2 \sin A \cos A \sin^2 B = 4 \sin A \sin^2 B \cos A$$

$$\Leftrightarrow \sin A \cos B + \sin B \cos A = 2 \sin B \cos A \text{ (do } \sin A > 0, \sin B > 0 \text{)}$$

$$\Leftrightarrow \sin A \cos B - \sin B \cos A = 0$$

$$\Leftrightarrow \sin(A - B) = 0$$

$$\Leftrightarrow A = B$$

Thay vào (2) ta được

$$\sin 2A = 2 \sin^2 A$$

$$\Leftrightarrow 2 \sin A \cos A = 2 \sin^2 A$$

$$\Leftrightarrow \cos A = \sin A \text{ (do } \sin A > 0 \text{)}$$

$$\Leftrightarrow \operatorname{tg} A = 1$$

$$\Leftrightarrow A = \frac{\pi}{4}$$

Do đó  $\Delta ABC$  vuông cân tại C

## V. TAM GIÁC ĐỀU

**Bài 221:** Chứng minh  $\Delta ABC$  đều nếu:

$$bc\sqrt{3} = R[2(b+c) - a] \text{ (*)}$$

$$\text{Ta có: (*)} \Leftrightarrow (2R \sin B)(2R \sin C)\sqrt{3} = R[2(2R \sin B + 2R \sin C) - 2R \sin A]$$

$$\Leftrightarrow 2\sqrt{3} \sin B \sin C = 2(\sin B + \sin C) - \sin(B + C)$$

$$\Leftrightarrow 2\sqrt{3} \sin B \sin C = 2(\sin B + \sin C) - \sin B \cos C - \sin C \cos B$$

$$\Leftrightarrow 2 \sin B \left[ 1 - \frac{1}{2} \cos C - \frac{\sqrt{3}}{2} \sin C \right] + 2 \sin C \left[ 1 - \frac{1}{2} \cos B - \frac{\sqrt{3}}{2} \sin B \right] = 0$$

$$\Leftrightarrow \sin B \left[ 1 - \cos \left( C - \frac{\pi}{3} \right) \right] + \sin C \left[ 1 - \cos \left( B - \frac{\pi}{3} \right) \right] = 0 \text{ (1)}$$

$$\text{Do } \sin B > 0 \text{ và } 1 - \cos\left(C - \frac{\pi}{3}\right) \geq 0$$

$$\sin C > 0 \text{ và } 1 - \cos\left(B - \frac{\pi}{3}\right) \geq 0$$

Nên vế trái của (1) luôn  $\geq 0$

$$\text{Do đó, (1)} \Leftrightarrow \begin{cases} \cos\left(C - \frac{\pi}{3}\right) = 1 \\ \cos\left(B - \frac{\pi}{3}\right) = 1 \end{cases}$$

$$\Leftrightarrow C = B = \frac{\pi}{3} \Leftrightarrow \Delta ABC \text{ đều.}$$

**Bài 222:** Chứng minh  $\Delta ABC$  đều nếu

$$\begin{cases} \sin B \sin C = \frac{3}{4} & (1) \\ a^2 = \frac{a^3 - b^3 - c^3}{a - b - c} & (2) \end{cases}$$

$$\text{Ta có: (2)} \Leftrightarrow a^3 - a^2b - a^2c = a^3 - b^3 - c^3$$

$$\Leftrightarrow a^2(b + c) = b^3 + c^3$$

$$\Leftrightarrow a^2(b + c) = (b + c)(b^2 - bc + c^2)$$

$$\Leftrightarrow a^2 = b^2 - bc + c^2$$

$$\Leftrightarrow b^2 + c^2 - 2bc \cos A = b^2 + c^2 - bc \text{ (do đl hàm cosin)}$$

$$\Leftrightarrow 2bc \cos A = bc$$

$$\Leftrightarrow \cos A = \frac{1}{2} \Leftrightarrow A = \frac{\pi}{3}$$

$$\text{Ta có: (1)} \Leftrightarrow 4 \sin B \sin C = 3$$

$$\Leftrightarrow 2[\cos(B - C) - \cos(B + C)] = 3$$

$$\Leftrightarrow 2[\cos(B - C) + \cos A] = 3$$

$$\Leftrightarrow 2 \cos(B - C) + 2\left(\frac{1}{2}\right) = 3 \quad \left(\text{do (1) ta có } A = \frac{\pi}{3}\right)$$

$$\Leftrightarrow \cos(B - C) = 1 \Leftrightarrow B = C$$

Vậy từ (1), (2) ta có  $\Delta ABC$  đều

**Bài 223:** Chứng minh  $\Delta ABC$  đều nếu:  

$$\sin A + \sin B + \sin C = \sin 2A + \sin 2B + \sin 2C$$

$$\begin{aligned} \text{Ta có: } \sin 2A + \sin 2B &= 2 \sin(A + B) \cos(A - B) \\ &= 2 \sin C \cos(A - B) \leq 2 \sin C \quad (1) \end{aligned}$$

$$\text{Dấu “=” xảy ra khi: } \cos(A - B) = 1$$

$$\text{Tương tự: } \sin 2A + \sin 2C \leq 2 \sin B \quad (2)$$

Dấu “=” xảy ra khi:  $\cos(A - C) = 1$

Tương tự:  $\sin 2B + \sin 2C \leq 2 \sin A$  (3)

Dấu “=” xảy ra khi:  $\cos(B - C) = 1$

Từ (1) (2) (3) ta có:  $2(\sin 2A + \sin 2B + \sin 2C) \leq 2(\sin C + \sin B + \sin A)$

$$\begin{aligned} \text{Dấu “=” xảy ra} &\Leftrightarrow \begin{cases} \cos(A - B) = 1 \\ \cos(A - C) = 1 \\ \cos(B - C) = 1 \end{cases} \Leftrightarrow A = B = C \\ &\Leftrightarrow \Delta ABC \text{ đều} \end{aligned}$$

**Bài 224:** Cho  $\Delta ABC$  có:

$$\frac{1}{\sin^2 2A} + \frac{1}{\sin^2 2B} + \frac{1}{\sin^2 2C} = \frac{1}{2 \cos A \cos B \cos C} (*)$$

Chứng minh  $\Delta ABC$  đều

Ta có: (\*)  $\Leftrightarrow \sin^2 2B \cdot \sin^2 2C + \sin^2 2A \sin^2 2C + \sin^2 2A \sin^2 2B$

$$= \frac{\sin 2A \cdot \sin 2B \cdot \sin 2C}{2 \cos A \cos B \cos C} \cdot (\sin 2A \sin 2B \sin 2C)$$

$$= 4 \sin A \sin B \sin C (\sin 2A \sin 2B \sin 2C)$$

$$\text{Mà: } 4 \sin A \sin B \sin C = 2 [\cos(A - B) - \cos(A + B)] \sin(A + B)$$

$$= 2 [\cos(A - B) + \cos C] \sin C$$

$$= 2 \sin C \cos C + 2 \cos(A - B) \sin(A + B)$$

$$= \sin 2C + \sin 2A + \sin 2B$$

Do đó, với điều kiện  $\Delta ABC$  không vuông ta có

$$(*) \Leftrightarrow \sin^2 2B \sin^2 2C + \sin^2 2A \sin^2 2C + \sin^2 2A \sin^2 2B$$

$$= \sin 2A \cdot \sin 2B \cdot \sin 2C (\sin 2A + \sin 2B + \sin 2C)$$

$$= \sin^2 2A \sin 2B \sin 2C + \sin^2 2B \sin 2A \sin 2C + \sin^2 2C \sin 2A \sin 2B$$

$$\Leftrightarrow \frac{1}{2} (\sin 2B \sin 2A - \sin 2B \sin 2C)^2 + \frac{1}{2} (\sin 2A \sin 2B - \sin 2A \sin 2C)^2$$

$$+ \frac{1}{2} (\sin 2C \sin 2A - \sin 2C \sin 2B)^2 = 0$$

$$\Leftrightarrow \begin{cases} \sin 2B \sin 2A = \sin 2B \sin 2C \\ \sin 2A \sin 2B = \sin 2A \sin 2C \\ \sin 2A \sin 2C = \sin 2C \sin 2B \end{cases}$$

$$\Leftrightarrow \begin{cases} \sin 2A = \sin 2B \\ \sin 2B = \sin 2C \end{cases} \Leftrightarrow A = B = C \Leftrightarrow \Delta ABC \text{ đều}$$

$$\Leftrightarrow \begin{cases} \sin 2A = \sin 2B \\ \sin 2B = \sin 2C \end{cases} \Leftrightarrow A = B = C \Leftrightarrow \Delta ABC \text{ đều}$$

**Bài 225:** Chứng minh  $\Delta ABC$  đều nếu:

$$\frac{a \cos A + b \cos B + c \cos C}{a \sin B + b \sin C + c \sin A} = \frac{2p}{9R} (*)$$

$$\begin{aligned}
\text{Ta có: } a \cos A + b \cos B + c \cos C &= 2R \sin A \cos A + 2R \sin B \cos B + 2R \sin C \cos C \\
&= R(\sin 2A + \sin 2B + \sin 2C) \\
&= R[2 \sin(A+B) \cos(A-B) + 2 \sin C \cos C] \\
&= 2R \sin C [\cos(A-B) - \cos(A+B)] = 4R \sin C \sin A \sin B
\end{aligned}$$

**Cách 1:**  $a \sin B + b \sin C + c \sin A$

$$= 2R(\sin A \sin B + \sin B \sin C + \sin C \sin A)$$

$$\geq 2R \sqrt[3]{\sin^2 A \sin^2 B \sin^2 C} \text{ (do bất Cauchy)}$$

$$\text{Do đó vế trái: } \frac{a \cos A + b \cos B + c \cos C}{a \sin B + b \sin C + c \sin A} \leq \frac{2}{3} \sqrt[3]{\sin A \sin B \sin C} \quad (1)$$

$$\text{Mà vế phải: } \frac{2p}{9R} = \frac{a+b+c}{9R} = \frac{2}{9}(\sin A + \sin B + \sin C)$$

$$\geq \frac{2}{3} \sqrt[3]{\sin A \sin B \sin C} \quad (2)$$

Từ (1) và (2) ta có

$$(*) \Leftrightarrow \sin A = \sin B = \sin C \Leftrightarrow \Delta ABC \text{ đều}$$

**Cách 2:** Ta có:  $(*) \Leftrightarrow \frac{4R \sin A \sin B \sin C}{a \sin B + b \sin C + c \sin A} = \frac{a+b+c}{9R}$

$$\Leftrightarrow \frac{4R \left(\frac{a}{2R}\right) \left(\frac{b}{2R}\right) \left(\frac{c}{2R}\right)}{a \left(\frac{b}{2R}\right) + b \left(\frac{c}{2R}\right) + \frac{ca}{2R}} = \frac{a+b+c}{9R}$$

$$\Leftrightarrow 9abc = (a+b+c)(ab+bc+ca)$$

Do bất đẳng thức Cauchy ta có

$$a+b+c \geq \sqrt[3]{abc}$$

$$ab+bc+ca \geq \sqrt[3]{a^2b^2c^2}$$

$$\text{Do đó: } (a+b+c)(ab+bc+ca) \geq 9abc$$

$$\text{Dấu = xảy ra } \Leftrightarrow a=b=c \Leftrightarrow \Delta ABC \text{ đều.}$$

**Bài 226:** Chứng minh  $\Delta ABC$  đều nếu

$$\cot gA + \cot gB + \cot gC = \operatorname{tg} \frac{A}{2} + \operatorname{tg} \frac{B}{2} + \operatorname{tg} \frac{C}{2} (*)$$

$$\text{Ta có: } \cot gA + \cot gB = \frac{\sin(A+B)}{\sin A \sin B} = \frac{\sin C}{\sin A \sin B}$$

$$\geq \frac{\sin C}{\left(\frac{\sin A + \sin B}{2}\right)^2} \text{ (do bất Cauchy)}$$

$$= \frac{2 \sin \frac{C}{2} \cos \frac{C}{2}}{\sin^2 \frac{A+B}{2} \cdot \cos^2 \frac{A-B}{2}} = \frac{2 \sin \frac{C}{2}}{\cos \frac{C}{2} \cos^2 \frac{A-B}{2}} \geq 2 \operatorname{tg} \frac{C}{2} \quad (1)$$

$$\text{Tương tự: } \cot gA + \cot gC \geq 2 \operatorname{tg} \frac{B}{2} \quad (2)$$

$$\cot gB + \cot gC \geq 2 \operatorname{tg} \frac{A}{2} \quad (3)$$

Từ (1) (2) (3) ta có

$$2(\cot gA + \cot gB + \cot gC) \geq 2 \left( \operatorname{tg} \frac{A}{2} + \operatorname{tg} \frac{B}{2} + \operatorname{tg} \frac{C}{2} \right)$$

Do đó dấu “=” tại (\*) xảy ra

$$\Leftrightarrow \begin{cases} \cos \frac{A-B}{2} = \cos \frac{A-C}{2} = \cos \frac{B-C}{2} = 1 \\ \sin A = \sin B = \sin C \end{cases}$$

$$\Leftrightarrow A = B = C$$

$$\Leftrightarrow \Delta ABC \text{ đều.}$$

## BÀI TẬP

1. Tính các góc của  $\Delta ABC$  biết:

$$a/ \cos A = \sin B + \sin C - \frac{3}{2} \quad (\text{ĐS: } B = C = \frac{\pi}{6}, A = \frac{2\pi}{3})$$

$$b/ \sin 6A + \sin 6B + \sin 6C = 0 \quad (\text{ĐS: } A = B = C = \frac{\pi}{3})$$

$$c/ \sin 5A + \sin 5B + \sin 5C = 0$$

2. Tính góc C của  $\Delta ABC$  biết:

$$a/ (1 + \cot gA)(1 + \cot gB) = 2$$

$$b/ \begin{cases} A, B \text{ nhọn} \\ \sin^2 A + \sin^2 B = \sqrt[3]{\sin C} \end{cases}$$

3. Cho  $\Delta ABC$  có:  $\begin{cases} \cos^2 A + \cos^2 B + \cos^2 C < 1 \\ \sin 5A + \sin 5B + \sin 5C = 0 \end{cases}$

Chứng minh  $\Delta$  có ít nhất một góc  $36^\circ$ .

4. Biết  $\sin^2 A + \sin^2 B + \sin^2 C = m$ . Chứng minh

a/  $m = 2$  thì  $\Delta ABC$  vuông

b/  $m > 2$  thì  $\Delta ABC$  nhọn

c/  $m < 2$  thì  $\Delta ABC$  tù.

5. Chứng minh  $\Delta ABC$  vuông nếu:

$$a/ \cos B + \cos C = \frac{b+c}{a}$$

$$b/ \frac{b}{\cos B} + \frac{c}{\cos C} = \frac{a}{\sin B \sin C}$$

$$c/ \sin A + \sin B + \sin C = 1 - \cos A + \cos B + \cos C$$

$$d/ \frac{(b-c)^2}{b^2} = \frac{2[1 - \cos(B-C)]}{1 - \cos 2B}$$

6. Chứng minh  $\Delta ABC$  cân nếu:

$$a/ \frac{1 + \cos B}{\sin B} = \frac{2a + c}{\sqrt{a^2 - c^2}}$$

$$b/ \frac{\sin A + \sin B + \sin C}{\sin A + \sin B - \sin C} = \cot g \frac{A}{2} \cdot \cot g \frac{B}{2}$$

$$c/ \operatorname{tg} A + 2\operatorname{tg} B = \operatorname{tg} A \cdot \operatorname{tg}^2 B$$

$$d/ a \left( \cot g \frac{C}{2} - \operatorname{tg} A \right) = b \left( \operatorname{tg} B - \cot g \frac{C}{2} \right)$$

$$e/ (p - b) \cot g \frac{C}{2} = \operatorname{ptg} \frac{B}{2}$$

$$f/ a + b = \operatorname{tg} \frac{C}{2} (a \operatorname{tg} A + b \operatorname{tg} B)$$

7.  $\Delta ABC$  là  $\Delta$  gì nếu:

$$a/ a \operatorname{tg} B + b \operatorname{tg} A = (a + b) \operatorname{tg} \frac{A + B}{2}$$

$$b/ c = c \cos 2B + b \sin 2B$$

$$c/ \sin 3A + \sin 3B + \sin 3C = 0$$

$$d/ 4S = (a + b - c)(a + c - b)$$

8. Chứng minh  $\Delta ABC$  đều nếu

$$a/ 2(a \cos A + b \cos B + c \cos C) = a + b + c$$

$$b/ 3S = 2R^2 (\sin^3 A + \sin^3 B + \sin^3 C)$$

$$c/ \sin A + \sin B + \sin C = 4 \sin A \sin B \sin C$$

$$d/ m_a + m_b + m_c = \frac{9R}{2} \text{ với } m_a, m_b, m_c \text{ là 3 đường trung tuyến}$$

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